Exercise 1 Solution

$f_X(x) = \begin{cases} \frac{1}{4}, & x \in (1,5) \\ 0, & x \notin (1,5) \end{cases}$, $f_Y(y) = \begin{cases} \frac{1}{4}, & y \in (1,5) \\ 0, & y \notin (1,5) \end{cases}$

because of the independence of X and Y the joint density is their product: $f_{X,Y}(x,y) = f_X(x) f_Y(y) = \begin{cases} \frac{1}{16}, & 1<x<5,\ 1<y<5 \\ 0, & \text{else} \end{cases}$

Thus, $P(X<Y) = \iint_{x<y} f_{X,Y}(x,y)\,dx\,dy = \int_1^5 \int_1^y \frac{1}{16}\,dx\,dy = \int_1^5 \frac{y}{16} - \frac{1}{16}\,dy =$

$= \frac{y^2}{32} - \frac{y}{16}\Big|_1^5 = \left(\frac{25}{32} - \frac{5}{16}\right) - \frac{1}{32} + \frac{1}{16} = \frac{1}{2}$

Exercise 2 Solution

X and Y have probability densities that are zero for negative values, this will hold for X+Y as well.

Now, we are using the convolution formula for $z \geq 0$ to get

$f_{X+Y}(z) = \int_{-\infty}^{\infty} f_X(x) f_Y(z-x)\,dx = \int_0^z f_X(x) f_Y(z-x)\,dx =$

$= \int_0^z 3xe^{-2x} \cdot 2(z-x)e^{-2(z-x)}\,dx = 6e^{-2z} \int_0^z xz - x^2\,dx =$

$= 6e^{-2z}\left(\frac{x^2}{2}z - \frac{x^3}{3}\Big|_0^z\right) = 6e^{-2z}\left(\frac{z^3}{2} - \frac{z^3}{3}\right) = z^3 e^{-2z}$

$f_{X+Y}(z) = \begin{cases} z^3 e^{-2z}, & \text{if } z \geq 0 \\ 0, & \text{otherwise} \end{cases}$

Exercise 3 Solution

By fact 7.9 the distribution of W is normal

with $\mu_W = 2\mu_X + 5\mu_Y = -3$

$\sigma_W^2 = 4\sigma_X^2 + 25\sigma_Y^2 = 112$