vrije Universiteit amsterdam

Studentnumber:
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School of Business and Economics

Exam:	Multivariate Econometrics
Code:	E_EORM_MVE
Examinator:	dr. H. Karabiyik
Co-reader:	dr. P. Gorgi
Date:	December 17, 2020
Time:	08:30
Duration:	2 hours and 45 minutes
Calculator allowed:	**Yes**
Graphical calculator allowed:	**No**
Scrap paper	**Yes**
Number of questions:	4 (where each question consists of multiple parts)
Type of questions:	Open
Answer in:	English

Remarks:

- Motivate all your answers.

Credit score:	A score of 100 points counts for a grade 10 for the exam. Each question is 25 points.
Grades:	The grades will be made public within 10 working days after the exam.
Inspection:	TBA
Number of pages:	11 (Including front page)

Good luck!

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Instructions

(i) All questions should be answered to get full points.

(ii) Each question is worth 25 points.

(iii) Read the instructions in the questions carefully.

(iv) Answer the questions as detailed as possible. Use mathematical expressions when necessary. You can use words when you cannot provide a formal mathematical answer to the questions.

(v) If a question is not clear to you, make your own assumptions to clarify the meaning of the question and then answer the question based on your assumptions.

(vi) See the back of this page for some standard results that you may make use of while answering the questions.

Some standard results

Suppose that the scalar process $\{z_t\}$ follows the following data generating process:

$$z_t = z_{t-1} + u_t,$$

where $z_0 = 0$ and u_t has the following properties:

(a) $u_t = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an $i.i.d$ sequence with mean zero and variance σ_ϵ^2, and finite fourth moment;

(b) σ^2 denotes the long run variance of $\{u_t\}$ and σ_u^2 denotes the contemporaneous variance of $\{u_t\}$.

Note that under these assumptions z_t can be written as a partial sum as

$$z_t = \sum_{s=1}^{t} u_s.$$

Let $W(r)$ be a standard Brownian motion process associated with u_t. Then the following results hold:

(1) $T^{-1/2} \sum_{t=1}^{T} u_t \xrightarrow{d} \sigma W(1)$;

(2) $T^{-1} \sum_{t=1}^{T} u_t^2 \xrightarrow{p} \sigma_u^2$;

(3) $T^{-1} \sum_{t=1}^{T} z_{t-1} u_t \xrightarrow{d} \frac{1}{2}\sigma^2 \left[W(1)^2 - \frac{\sigma_u^2}{\sigma^2} \right]$;

(4) $T^{-3/2} \sum_{t=1}^{T} t u_{t-j} \xrightarrow{d} \sigma \left\{ W(1) - \int_0^1 W(r) dr \right\}$ for $j = 0, 1, \ldots$;

(5) $T^{-3/2} \sum_{t=1}^{T} z_{t-1} \xrightarrow{d} \sigma \int_0^1 W(r) dr$;

(6) $T^{-2} \sum_{t=1}^{T} z_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 W(r)^2 dr$;

(7) $T^{-5/2} \sum_{t=1}^{T} t z_{t-1} \xrightarrow{d} \sigma \int_0^1 r W(r) dr$;

(8) $T^{-3} \sum_{t=1}^{T} t z_{t-1}^2 \xrightarrow{d} \sigma^2 \int_0^1 r W(r)^2 dr$;

(9) $T^{-(v+1)} \sum_{t=1}^{T} t^v \rightarrow 1/(v+1)$ for $v = 0, 1, \ldots$;

(10) Suppose that the DGP of another time series process y_t follows the model

$$y_t = y_{t-1} + e_t,$$

where $y_t = 0$ and e_t satisfy the same assumptions as **(a)** and has a long run variance σ_e^2 and $W_e(r)$ is a standard Brownian motion processs associated with e_t, then

$$T^{-2} \sum_{t=1}^{T} z_t y_t \xrightarrow{d} \sigma \sigma_e \int_0^1 W(r) W_e(r) dr.$$

Question 1: Conceptual Questions (25 points out of 100 points)

Below you will find **3** statements. All these are related to the concepts/techqniques that have been discussed during the lectures. Some of these statements are correct, some are wrong, some need further clarification. You need to provide a brief, to the point answer that would contain **(i) short explanations/definitions of the concepts mentioned in the statement, (ii) your judgement about the statement about whether it is correct/wrong/unclear/incomplete, and an explanation of your judgement (iii) a correction of the statement.** The concepts that you need to explain and define are written in *italics*. A formal answer using mathematics is possible, sometimes very useful but not always necessary.

(a) (5 points) *Reduced form* representation of a VAR model can be uniquely converted to a *structural form* VAR model. It is always preferable to consider reduced from VARs.

(b) (5 points) Suppose that y_t and x_t are two $I(1)$ *processes* and these two processes are *cointegrated* with the cointegrating vector $(1, -\beta)$. Here, β can be estimated consistently by estimating the regression model

$$y_t = \beta x_t + u_t.$$

Furthermore the hypothesis of $\mathcal{H}_0 : \beta = 1$ can be tested by using normal distribution.

(c) (15 points) For a panel data variable $y_{i,t}$, a dynamic panel data model can be written as

$$y_{i,t} = \rho y_{i,t-1} + u_{i,t}.$$

Let N represent the number of cross-section units and T represent the number of time series periods. Suppose that $u_{i,t}$ follows

$$u_{i,t} = \mu_i + \epsilon_{i,t},$$

where $\mu_i \sim (0, \sigma_\mu^2)$ and $\epsilon_{i,t} \sim (0, \sigma_\epsilon^2)$. This is called the *interactive fixed effects* assumption for $u_{i,t}$. When T is fixed as $N \to \infty$, the *fixed effects estimator* of ρ is unbiased and consistent.

Question 2: Modeling and stationarity (25 points out of 100 points)

(a) (15 points) Consider two processes y_t and x_t with the DGPs

$$y_t = A_1 y_{t-1} + B_1 y_{t-2} + A_2 x_{t-1} + u_{y,t},$$
$$x_t = A_3 y_{t-1} + A_4 x_{t-1} + u_{x,t}.$$

Suppose that

$$u_{y,t} = C_1 u_{y,t-1} + \varepsilon_{1,t},$$
$$u_{x,t} = \varepsilon_{1,t} + \varepsilon_{2,t},$$

where for $\boldsymbol{\varepsilon}_t = (\varepsilon_{1,t}, \varepsilon_{2,t})'$.

$$\boldsymbol{\varepsilon}_t = \begin{pmatrix} \varepsilon_{1,t} \\ \varepsilon_{2,t} \end{pmatrix} \sim IN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_1^2 & 0 \\ 0 & \sigma_2^2 \end{pmatrix}\right],$$

Now we define

$$\mathbf{w}_t = \begin{pmatrix} y_t \\ x_t \end{pmatrix}.$$

(i) Write the two models for y_t and x_t given above in the form of a VAR(2) model for $\mathbf{w}_t$.

(ii) Show that for $\mathbf{w}_t$ we can obtain a model in the form

$$\Delta \mathbf{w}_t = \mathbf{E}\mathbf{w}_{t-1} + \mathbf{F}\Delta \mathbf{w}_{t-1} + \mathbf{G}\Delta \mathbf{w}_{t-2} + \mathbf{H}\boldsymbol{\varepsilon}_t,$$

where

$$\mathbf{H} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}.$$

Write $\mathbf{E}$, $\mathbf{F}$ and $\mathbf{G}$ in terms of the model parameters A_1, A_2, A_3, A_4, B_1 and C_1.

(iii) Briefly discuss under what conditions at least one of the elements of $\mathbf{w}_t$ is I(2)?

(b) (10 points) Consider the bivariate system for y_t and x_t. Let $\mathbf{w}_t = (y_t, x_t)'$. The DGP for $\mathbf{w}_t$ can be written as

$$\Delta \mathbf{w}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{w}_{t-1} + \boldsymbol{\Gamma}\Delta \mathbf{w}_{t-1} + \boldsymbol{\epsilon}_t,$$

where

$$\boldsymbol{\alpha} = \begin{pmatrix} \alpha_1 \\ 0 \end{pmatrix}, \quad \boldsymbol{\beta} = \begin{pmatrix} 1 \\ -\beta_1 \end{pmatrix},$$

and

$$\boldsymbol{\Gamma} = \begin{pmatrix} 0 & \gamma_1 \\ 0 & \gamma_2 \end{pmatrix}$$

and

$$\boldsymbol{\epsilon}_t = \begin{pmatrix} \epsilon_{y,t} \\ \epsilon_{x,t} \end{pmatrix} \sim IN\left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_y^2 & 0 \\ 0 & \sigma_x^2 \end{pmatrix}\right].$$

Show that $\boldsymbol{\beta}'\mathbf{w}_t$ is weakly stationary if $|\gamma_2| < 1$, and $-2 < \alpha_1 < 0$. Write down the additional necessary conditions (if any). In addition to your derivations explain with your own words why do we need the conditions $|\gamma_2| < 1$, and $-2 < \alpha_1 < 0$ and what happens if these conditions are not satisfied.

Question 3: Asymptotic Derivations (25 points out of 100 points)

(a) (15 points) **A nonparametric test for unit roots:**

Suppose that we have the following data generating processes (DGP) for $\{y_t\}$

$$y_t = \rho y_{t-1} + u_t,$$

for $t = 1, \ldots, T$. We assume:

- $u_t = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an *i.i.d* sequence with mean zero and variance σ_ϵ^2, and finite fourth moment;
- σ^2 denotes the long run variance of $\{u_t\}$ and σ_u^2 denotes the contemporaneous variance of $\{u_t\}$;
- $y_0 = 0$.

We would like to test for unit roots. So we are interested in testing

$$\mathcal{H}_0 : \ \rho = 1,$$

against the alternative

$$\mathcal{H}_1 : \ |\rho| < 1.$$

For this reason, we consider the test statistic

$$VR = \frac{T\left(\sum_{t=1}^{T} \Delta y_t\right)^2}{\sum_{t=1}^{T} y_t^2}$$

(i) Derive carefully the order of probability and the limiting distribution of VR under the null hypothesis.

(ii) Obtain the order of probability of VR under the alternative hypothesis.

(iii) What do you conclude about the consistency of this test?

(b) (10 points) Suppose that we have the following data generating processes (DGP) for $\{y_t\}$ and $\{x_t\}$

$$y_t = \delta + y_{t-1} + u_{y,t}$$
$$x_t = \mu + u_{x,t}$$

for $t = 1, \ldots, T$. We assume:

- $u_{y,t} = \psi(L)\epsilon_t = \sum_{j=0}^{\infty} \psi_j \epsilon_{t-j}$ where $\sum_{j=0}^{\infty} j \cdot |\psi_j| < \infty$ and $\{\epsilon_t\}$ is an $i.i.d$ sequence with mean zero and variance σ_ϵ^2, and finite fourth moment. σ_y^2 denotes the long run variance of $\{u_{y,t}\}$ and $\sigma_{u,y}^2$ denotes the contemporaneous variance of $\{u_{y,t}\}$;
- $u_{x,t} \sim i.i.d(0, \sigma_{u,x}^2)$;
- $y_0 = 0$;

We consider the estimation of the regression model

$$y_t = \beta x_t + error,$$

using a sample of T observations.

Consider the least squares estimator

$$\widehat{\beta} = \frac{\sum_{t=1}^{T} y_t x_t}{\sum_{t=1}^{T} x_t^2}.$$

(i) Derive and discuss the orders of probability and limiting distributions of the numerator and the denominator of $\widehat{\beta}$. Write down any necessary additional assumptions.

(ii) Use your findings for **(i)** to comment on the asymptotic behaviour of $\hat{\beta}$. What is wrong in this analysis?

Question 4: Empirical Application (25 points out of 100 points)

(a) (10 points) Samwise is an econometrics student from Rohan University. He wants to analyze the evolution of average atmospheric temperature and gross domestic product over time and use his findings to investigate the relation between these variables. He collects annual data on these variables from two countries for the period 1920 - 2020. These countries are called Gondor and Isengard. The retained variables are

- log of average annual atmpospheric temperature of Gondor ($temp_{g,t}$),
- log of gross domestic product of Gondor ($gdp_{g,t}$)
- log of average annual atmpospheric temperature of Isengard ($temp_{i,t}$)
- log of gross domestic product of Isengard ($gdp_{i,t}$)

He first tests for unit roots by using the Dickey-Fuller test for the null hypothesis of unit root against alternative hypothesis of trend stationarity.

Variables	DF-statistic	5 % $t-$distribution critical values	5% Dickey-fuller critical values without trend	5% Dickey-fuller critical values with trend
$temp_{g,t}$	-1.89	-1.64	-2.89	-3.45
$gdp_{g,t}$	-1.67	-1.64	-2.89	-3.45
$temp_{i,t}$	-2.56	-1.64	-2.89	-3.45
$gdp_{i,t}$	-3.32	-1.64	-2.89	-3.45

Given this unit root test output he wants to estimate a vector error correction model for the 4 variables that he has data on.

(i) Comment on the results of the unit root tests. Discuss whether the choice of the unit root test is justifiable. Do you have any recommendations to Samwise that would improve his unit root analysis? If yes, what are they? Discuss in detail.

(ii) Given the unit root test results, what are your suggestions for the VECM model that Samwise wants to construct and analyze? If he wants to discover the potential cointegrating relations between the 4 variables, how should he proceed? Explain the steps he needs to follow in detail.

(b) (15 points) Another econometrics student from Rohan University named Frodo is analyzing a panel data set of daily wind speed and insurance claims for 150 towns over 360 days. Let $w_{i,t}$ denote the average wind speed in town i on day t and $c_{i,t}$ denote the total daily amount of insurance claims made from town i on day t. Frodo considers the model

$$c_{i,t} = \beta_i w_{i,t} + u_{i,t}.$$

He suspects that there might be correlation between the error terms of the models for wind speeds of different towns, such that

$$Cov(u_{i,t} u_{j,t}) \neq 0, \text{for } i \neq j.$$

He starts looking for an advice on how to analyze this panel data set.

(i) One of the ways to model this correlation is to assume a "spatial" model. Another way is to assume the existence of an "unobserved common factor". Explain to Frodo how these two approaches can be used model to the correlation between $u_{i,t}$ and $u_{j,t}$ for $i \neq j$.

(ii) Assume that there is an unobserved factor that affects $u_{i,t}$ and $w_{i,t}$ for all $i = 1, \ldots, N$. Explain to Frodo why it is not a good idea to ignore this factor structure even if he wants to estimate the individual specific coefficients, β_i by using OLS.

(iii) Now assume that β_i follows the random coefficient model, such that

$$\beta_i = \beta + \nu_i,$$

where $\nu_i \sim i.i.d(0, \omega_\nu)$, for $i = 1, \ldots, N$ and $|\beta| < K < \infty$, $0 < \omega_\nu < \infty$. In the presence of unobserved common factors, explain to Frodo, how he can use Pesaran's Common-Correlated-Effects (CCE) estimator to obtain a consistent estimator for β_i and for β.